KNT/KW/16/5109

Bachelor of Science (B.Sc.) Semester–III (C.B.S.) Examination

MATHEMATICS (M_6 Differential Equations and Group Homomorphism)

Paper–II

Time : Three Hours] [Maximum Marks : 60

N.B. :— (1) Solve all **five** questions.

(2) All questions carry equal marks.

(3) Question Nos. **1** to **4** have an alternative. Solve each question in full or its alternative in full.

UNIT–I

1. (A) Prove the recurrence relation :

$$2nJ_n(x) = x\left[J_{n-1}(x) + J_{n+1}(x)\right].$$ 6

(B) Prove that :

(i) $J_{-3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(-\frac{\cos x}{x} - \sin x\right)$, (ii) $J_{3/2}(x) = \sqrt{\frac{2}{\pi x}}\left(\frac{\sin x}{x} - \cos x\right)$. 6

OR

(C) Prove that the Legendre's polynomial $P_n(x)$ is the coefficient of h^n in the ascending power series expansion of $(1 - 2xh + h^2)^{-1/2}$, $|x| \le 1$, $|h| < 1$. 6

(D) Prove that $\int_{-1}^{1} P_m(x)P_n(x)\, dx = \frac{2}{2n+1}$; if $m = n$. 6

UNIT–II

2. (A) If L[f(t)] = F(s), then prove that :

(i) $L[e^{at} f(t)] = F(s - a)$ and

(ii) $L[e^{-at} f(t)] = F(s + a)$.

Hence show that :

$$L\left[e^{at} \cos bt\right] = \frac{s-a}{(s-a)^2 - b^2}.$$ 6

(B) Find the inverse Laplace transform of $\frac{s^2 - s - 2}{s(s+3)(s-2)}$. 6

OR

(C) If L[f(t)] = F(s), then prove that

$$L\left[\int_0^t f(u)\, du\right] = \frac{F(s)}{s}.$$

Hence find $L\left[\int_0^t \frac{\sin u}{u}\, du\right]$. 6

(D) By using convolution theorem, evaluate :

$$L^{-1}\left[\frac{s}{(s^2+4)^2}\right].$$ 6

UNIT–III

3. (A) Solve $y''' + 2y'' - y - 2y = 0$, given that $y(0) = y'(0) = 0$. 6

(B) Solve $y'' + ty' - y = 0$, given that $y(0) = 0$, $y'(0) = 2$. 6

OR

(C) Solve $\frac{\partial u}{\partial t} = 3\frac{\partial^2 u}{\partial x^2}$, given that $u(0, t) = 0$, $u(5, t) = 0$, $u(x, 0) = \sin \pi x$. 6

(D) Find the Fourier sine transform of $\frac{e^{-\lambda x}}{x}$, $\lambda > 0$. 6

UNIT–IV

4. (A) Prove that a subgroup N of a group G is a normal subgroup of G if and only if each left coset of N in G is a right coset of N in G. 6

(B) Let G be a group and N be its normal subgroup. Then prove that the set $\frac{G}{N}$ of all cosets is a group with respect to multiplication of cosets as $N_a . N_b = N_{ab}$. 6

OR

(C) Let $f : G \to G'$ be a homomorphism of a group G onto a group G' and K be the Kernel of f. Then prove that $G/K \cong G'$. 6

(D) Let G be the additive group of real numbers and $G' = G$. Show that a mapping $f : G \to G'$ defined by $f(x) = 12x$, $\forall x \in G$ is a homomorphism. Is it 1 – 1 and onto ? Also find Kernel K of homomorphism f. 6

QUESTION–V

5. (A) Show that $(xJ_1)' = xJ_0$. 1½

(B) Show that $P_n(1) = 1$. 1½

(C) Find $L\, e^{-4t} (\sin 5t + 3 \cos 2t)$. 1½

(D) Evaluate $L \{t . \sinh 2t\}$. 1½

(E) Solve $\frac{dx}{dt} + x = 0$, $x(0) = 2$. 1½

(F) Find the Fourier cosine transform of the function

$$f(x) = \begin{cases} 1 & , \quad 0 < x < a \\ 0 & , \quad x \geq a \end{cases}$$ 1½

(G) Prove that every cyclic group is abelian. 1½

(H) If $G = \{1, -1, i, -i\}$ is a group under multiplication and I, the additive group of all integers. Show that the function $f : I \to G$ defined by $f(n) = i^n$, $\forall n \in I$, is a homomorphism. 1½